Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Advanced

Thursday 12 June 2008 – Morning
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1.

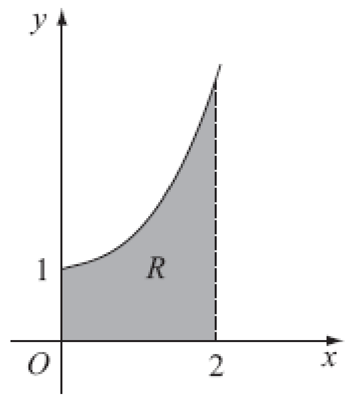

Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{e}^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$\mathrm{e}^{0.08}$		$\mathrm{e}^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

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2. (a) Use integration by parts to find $\int x\mathrm{e}^x \, \mathrm{d}x$.

(3)

(b) Hence find $\int x^2\mathrm{e}^x \, \mathrm{d}x$.

(3)

3.

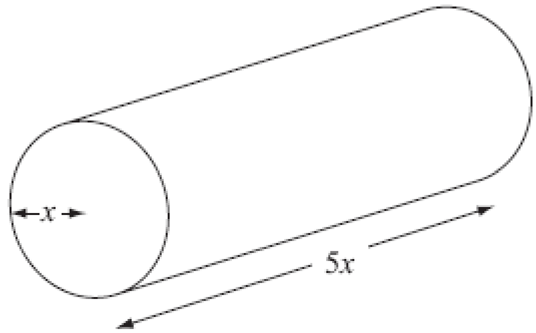

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\,\text{s}^{-1}$.

(*a*) Find $\dfrac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(*b*) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

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4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(*a*) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(*b*) Find the coordinates of P and Q.

(3)

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5. (*a*) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(*b*) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

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6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(*b*) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(*c*) Show that A lies on l_1.

(1)

The point B is the image of A after reflection in the line l_2.

(*d*) Find the position vector of B.

(3)

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7. (*a*) Express $\dfrac{2}{4 - y^2}$ in partial fractions.

(3)

(*b*) Hence obtain the solution of

$$2 \cot x \frac{\mathrm{d}y}{\mathrm{d}x} = (4 - y^2)$$

for which $y = 0$ at $x = \dfrac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$.

(8)

8.

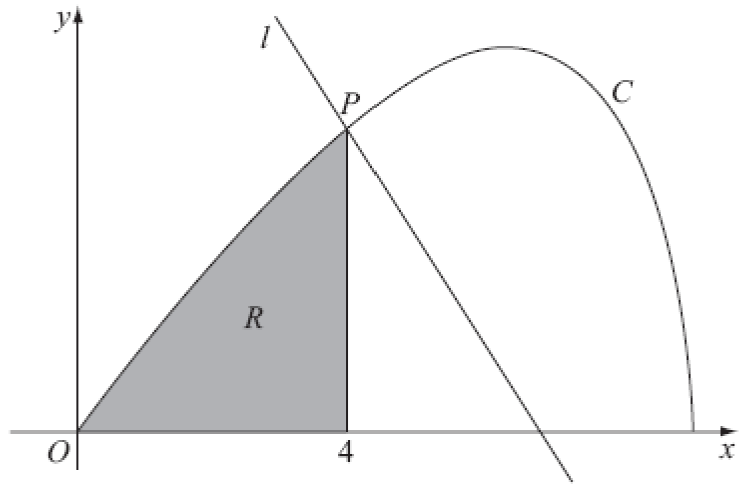

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \le t \le \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \, \mathrm{d}t$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (*a*)	<table><tr><td>x</td><td>0</td><td>0.4</td><td>0.8</td><td>1.2</td><td>1.6</td><td>2</td></tr><tr><td>y</td><td>e^0</td><td>$e^{0.08}$</td><td>$e^{0.32}$</td><td>$e^{0.72}$</td><td>$e^{1.28}$</td><td>e^2</td></tr><tr><td>or y</td><td>1</td><td>1.08329…</td><td>1.37713…</td><td>2.05443…</td><td>3.59664…</td><td>7.38906…</td></tr></table>	B1 (1)
	Area $\approx \frac{1}{2}\times 0.4\,;\times\underline{\left[\ e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\ \right]}$	B1; M1
	$= 0.2\times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf)	A1 (3)
		(4 marks)
2. (a)	$\begin{Bmatrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{Bmatrix}$	
	$\int xe^x\ dx = xe^x - \int e^x.1\,dx$	M1 A1
	$= xe^x - \int e^x\ dx$	
	$= xe^x - e^x\ (+c)$	A1 (3)
(*b*)	$\begin{Bmatrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{Bmatrix}$	
	$\int x^2e^x\ dx = x^2e^x - \int e^x.2x\,dx$	M1 A1
	$= x^2e^x - 2\int xe^x\,dx$	
	$= x^2e^x - 2\left(xe^x - e^x\right) + c$	A1 (3)
		(6 marks)

Question Number	Scheme	Marks
3. (*a*)	From question, $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.032$	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}x} = \right\} 2\pi x$	B1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}t} \div \frac{\mathrm{d}A}{\mathrm{d}x} = (0.032)\frac{1}{2\pi x}$; $\left\{= \frac{0.016}{\pi x}\right\}$	M1
	When $x = 2\,\mathrm{cm}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{0.016}{2\pi}$	
	Hence, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.002546479...$ $(\mathrm{cm\ s^{-1}})$	A1 cso (4)
(*b*)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}x} = 15\pi x^2$	B1 ft
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right)$; $\{= 0.24x\}$	M1
	When $x = 2\,\mathrm{cm}$, $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.24(2) = \underline{0.48}$ $(\mathrm{cm^3\,s^{-1}})$	A1 (4)
		(8 marks)
4. (*a*)	$3x^2 - y^2 + xy = 4$ (eqn $*$)	
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{\times}\right\}$ $\underline{6x - 2y\frac{\mathrm{d}y}{\mathrm{d}x}} + \left(\underline{\underline{y + x\frac{\mathrm{d}y}{\mathrm{d}x}}}\right) = \underline{0}$	M1 B1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	M1
	giving $-18x - 3y = 8x - 16y$	
	giving $13y = 26x$	M1
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	A1 cso (6)
(*b*)	At P & Q, $y = 2x$. Substituting into eqn $*$	
	gives $3x^2 - (2x)^2 + x(2x) = 4$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	A1
	$y = 2x \Rightarrow y = \pm 4$, hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$	A1 (3)
		(9 marks)

Question Number	Scheme	Marks
5. (*a*)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	B1
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})(**x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(**x)^2+\ldots}\right]$ **with** $** \neq 1$	M1; A1 ft
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+\ldots}\right]$	
	$= \frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+\ldots\right]$	A1 A1 (5)
(*b*)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\ldots\right)$	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+\ldots\ldots$ $+\,4+\frac{3}{2}x+\frac{27}{32}x^2+\ldots\ldots$	M1
	$= 4+2x;+\frac{33}{32}x^2+\ldots$	A1; A1 (4)
		(9 marks)

Question Number	Scheme	Marks
6. (*a*)	Lines meet where: $\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$	
	Any two of $\mathbf{i}:\ -9+2\lambda=3+3\mu$ (1); $\mathbf{j}:\ \lambda=1-\mu$ (2); $\mathbf{k}:\ 10-\lambda=17+5\mu$ (3)	M1
	(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	M1
	(2) gives: $\lambda=1--2=3$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	M1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	A1
	Either check **k:** $\lambda=3:$ LHS $=10-\lambda=10-3=7$; $\mu=-2:$ RHS $=17+5\mu=17-10=7$	B1 (6)
(*b*)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$	
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	M1 A1 (2)
	Then l_1 is perpendicular to l_2.	
(*c*)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$	
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}$. Hence the point A lies on l_1.)	B1 (1)
(*d*)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection	
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1 ft
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$	

Question Number	Scheme	Marks
	$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$	M1 ft
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	A1
		(12 marks)
7. (*a*)	$\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ so $2 \equiv A(2+y) + B(2-y)$	M1
	Let $y = -2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$, Let $y = 2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	M1
	giving $\underline{\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}}$	A1 cao (3)
(*b*)	$\int \frac{2}{4-y^2}\,\mathrm{d}y = \int \frac{1}{\cot x}\,\mathrm{d}x$	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,\mathrm{d}y = \int \tan x\,\mathrm{d}x$	
	$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) \ + (c)$	B1 M1 A1 ft
	$y = 0,\ x = \frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right) + c$	M1
	$\left\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\right\}$	
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$	
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$	
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	M1
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$	
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 (8)
		(11 marks)

Question Number	Scheme	Marks
8. (*a*)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
(*b*)	$x = 8\cos t, \quad y = 4\sin 2t$	
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	
	Hence m(**N**) $= -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	M1
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
(*c*)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$	
	$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$ (*)	A1 (4)
(*d*)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
		(16 marks)